The region of a fracture surface that formed during stage II propagation is characterized by two types of markings termed

*beachmarks*

and

*striations.*
Beachmarks (clamshell marks) are of macroscopic dimensions and are found in components that have experienced interruptions during stage II propagation.

Fracture surface of a rotating steel shaft that experienced fatigue failure.
On the other hand, **fatigue striations** are microscopic in size and subject to observation with the electron microscope (either TEM or SEM).

Transmission electron fractograph showing fatigue striations in aluminum.
Each striation is thought to represent the advance distance of the crack front during a single load cycle.

Striation width depends on, and increases with, increasing stress range.

The presence of beachmarks and/or striations on a fracture surface confirms that the cause of failure was fatigue. Nevertheless, the absence of either or both does not exclude fatigue as the cause of failure.
Beachmarks and striations will not appear on that region over which the rapid failure occurs.
When cracks initiate at the surface of a component on two closely spaced planes, the cracks grow and eventually join to form a ratchet mark.

Radial marks, on the other hand, radiate from the fracture origin and are visible to the unaided eye or at low magnification.
Schematic of marks on surfaces of fatigue fractures produced in smooth and notched components with round cross sections under various loading conditions at high and low nominal stress.

Crack Propagation Rate

Even though measures may be taken to minimize the possibility of fatigue failure, cracks and crack nucleation sites will always exist in structural components. Under the influence of cyclic stresses, cracks will inevitably form and grow; this process, if unabated, can ultimately lead to failure.

The intent of the present discussion is to develop a criterion whereby fatigue life may be predicted on the basis of material and stress state parameters.
Experimental techniques are available for monitoring crack length during the cyclic stressing. Data are recorded and plotted as crack length $a$ vs. number of cycles $N$. 

$$\sigma_2 > \sigma_1$$

$$(\frac{da}{dN})_{a_1, \sigma_1}$$

$$(\frac{da}{dN})_{a_1, \sigma_2}$$

Crack length $a$

Cycles $N$
The stage II crack growth rate is a function of not only stress level and crack size but also material variables. This rate may be expressed in terms of the stress intensity factor $K$ and takes the form:

$$\frac{da}{dN} = A\Delta K^m$$  \hspace{1cm} \text{Paris-Erdogan Law}$$

$A$ and $m$ are constants for the particular material, which will also depend on environment, frequency, and the stress ratio.
\( \Delta K \) is the stress intensity factor range at the crack tip, i.e.

\[
\Delta K = K_{\text{max}} - K_{\text{min}}
\]

or from fracture mechanics:

\[
\Delta K = Y \Delta \sigma \sqrt{\pi a} = Y \left( \sigma_{\text{max}} - \sigma_{\text{min}} \right) \sqrt{\pi a}
\]

The typical fatigue crack growth rate behavior of materials is represented schematically as the logarithm of crack growth rate \( da/dN \) vs. the logarithm of the stress intensity factor range \( \Delta K \).
Fatigue crack growth rate, \( \frac{da}{dN} \) (log scale)

Region I
Non-propagating fatigue cracks

Region II
Linear relationship between \( \log \Delta K \) and \( \log \frac{da}{dN} \)

Region III
Unstable crack growth

\[
\frac{da}{dN} = A\Delta K^m
\]
The curve is essentially linear in Region II.

\[ \frac{da}{dN} = A \Delta K^m \]

\[ \log \left( \frac{da}{dN} \right) = \log \left[ A (\Delta K)^m \right] \]

\[ \log \left( \frac{da}{dN} \right) = m \log \Delta K + \log A \]
Design Against Fatigue

One of the goals of failure analysis is to be able to predict fatigue life for some component, given its service constraints and laboratory test data. We are now able to develop an analytical expression for $N_f$, due to stage II, by integration of the Paris-Erdogan equation.

\[
dN = \frac{da}{A\Delta K^m}
\]

Integrating:

\[
N_f = \int_{0}^{N_f} dN = \int_{a_0}^{a_c} \frac{da}{A\Delta K^m}
\]

\[
N_f = \int_{a_0}^{a_c} \frac{da}{A(Y\Delta \sigma \sqrt{\pi a})^m}
\]

\[
N_f = \frac{1}{A\pi^{m/2}(\Delta \sigma)^m} \int_{a_0}^{a_c} \frac{da}{Y^m a^{m/2}}
\]
A relatively large sheet of steel is to be exposed to cyclic tensile and compressive stresses of magnitudes 100 MPa and 50 MPa, respectively. Prior to testing, it has been determined that the length of the largest surface crack is 2.0 mm. Estimate the fatigue life of this sheet if its plane strain fracture toughness is 25 MPa$\sqrt{\text{m}}$ and the values of $m$ and $A$ in the Paris-Erdoğan equation are 3.0 and $1.0 \times 10^{-12}$, respectively, for $\Delta \sigma$ in MPa and $a$ in m.
Solution
First we need to find the upper limit of the integral for the Paris-Erdoğan equation. Assuming a stress of 100 MPa in the fracture toughness equation,

\[ a_c = \frac{1}{\pi} \left( \frac{K_{lc}}{\sigma Y} \right)^2 = \frac{1}{\pi} \left( \frac{25 \text{ MPa}\sqrt{m}}{(100 \text{ MPa})(1)} \right)^2 = 0.02 \text{ m} \]

\[ N_f = \frac{1}{A\pi^{m/2}(\Delta\sigma)^m} \int_{a_0}^{a_c} \frac{da}{Y^m a^{m/2}} \]

\[ = \frac{1}{A\pi^{3/2}(\Delta\sigma)^3} \int_{a_0}^{a_c} \frac{da}{Y^3 a^{3/2}} \]
\[ N_f = \frac{1}{A \pi^{3/2} (\Delta \sigma)^3 Y^3} (-2a^{1/2}) \]

\[ = \frac{2}{A \pi^{3/2} (\Delta \sigma)^3 Y^3} \left( \frac{1}{\sqrt{a_0}} - \frac{1}{\sqrt{a_c}} \right) \]

\[ = \frac{2}{(1.0 \times 10^{-12}) \pi^{3/2} (100)^3 1^3} \left( \frac{1}{\sqrt{0.002}} - \frac{1}{\sqrt{0.02}} \right) \]

\[ = 5.49 \times 10^6 \text{ cycles} \]